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Research Note

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Regulation, Precision and Benchmarks

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Abstract

In financial markets, dealers may take advantage of information asymmetries and extract a rent from buy-side traders. We show that an increase in the precision of a benchmark reduces noise in market prices and increases expected demands by overcoming traders' and regulators' inability to penalize dealers sufficiently. Regulations that increase precision can therefore have positive effects on the overall market.

1 Introduction

Many industries, but particularly the financial services industry, use benchmarks to settle contracts, monitor trade executions, and signal the prices available in the market. However, until very recently, benchmarks were not subject to any regulation. This changed in 2013 when the Financial Conduct Authority started regulating the London InterBank Offered Rate (LIBOR) and subsequently in 2015 when it started regulating seven additional benchmarks. Since 2013, many international organizations such as IOSCO and the Financial Stability Board have developed guidance and rules on benchmarks, and the EU Benchmark Regulation entered into force in January 2018. So, many interventions have taken place in this area.

Recent theoretical work by Duffie, Dworczak, and Zhu (2017) shows that the existence of a benchmark improves the matching process in over-the-counter markets and can increase social welfare under conditions. When discussing welfare effects Duffie, Dworczak, and Zhu (2017) compare the case where a benchmark exists to the case where it does not. In this paper we focus on understanding the theoretical impact of an increase in the "quality" of the benchmark, since this is the likely result of the aforementioned regulations. We model the regulatory intervention as a reduction in noise in the benchmark fixing process and hence an increase in precision of the benchmark. Our results imply that effective regulation of a benchmark can reduce noise in market prices and increase expected demands, providing a theoretical rationale for many of the recent regulatory interventions in this area.

In our model, traders cannot observe dealers' marginal costs but, as in Duffie, Dworczak, and Zhu (2017), they can observe a public signal aggregating the information (the benchmark). Differently from Duffie, Dworczak, and Zhu (2017) the benchmark is measured with noise in our setting. The noise represents traders' different interpretations of the same signal (because of a lack of precision in the benchmark fixing) and imperfections in the benchmark assessment by dealers themselves (because of a lack of quality in the production costs data).

Due to the information asymmetry between dealers and traders, traders have to pay more than the efficient cost, and this reduces their welfare. To solve this problem, traders and regulators can decide to "punish" the dealers if the benchmark realization shows they are taking advantage of their position (by charging a high price). However, penalties are limited: traders can only decide not to buy from the dealers, and the regulatory fines necessary to restore the optimal allocation may be too high to be practically implemented.

The constraints preclude the implementation of the optimal outcome, which is for traders to pay a price equal to dealers' true cost of production. We show that a policy that reduces the noise in the benchmark fixing process overcomes these limitations and restores the optimal outcome. In practice, such a policy would be very similar to the imposition of systems and controls that was central to the rules introduced by the FCA in 2013 and extended to other benchmarks in 2015.

2 The model

2.1 Structure of the model

We start with a market with risk neutral dealers and traders. As in Duffie, Dworczak, and Zhu (2017), n dealers sell a homogeneous good to a continuum of traders who differ in search costs. The timing of the game is as follows:

- Nature draws dealers' marginal costs, traders' search costs, and the benchmark realization;
- Dealers move first and set the price of the good;
- Traders observe the prices in the market and the benchmark realization, and decide whether to enter the market.

2.2 The benchmark

A trader either buys one unit of good and pays price p_i to Dealer i, or stays outside the market. Each dealer supplies the same good from the wholesale market and has a cost of production a_i for each unit. Productions costs, which are also marginal costs, are heterogeneous and reflect dealers' efficiency. A dealer with a low a is more efficient than a dealer with a high a. Moreover, each dealer only knows her own marginal cost.

Traders cannot observe dealers' marginal costs, but they use the benchmark y to observe with noise the average cost of production among dealers. The benchmark y is therefore defined as

$$y = \sum_{i=1}^{n} \frac{a_i}{n} + \epsilon \tag{1}$$

where $\epsilon \sim F(0, \sigma^2)$ is the noise component, with density f and cumulative distribution F. As $\sigma \to 0$, the benchmark becomes more precise, so the noise represents the accuracy of the benchmark fixing.

2.3 Quantities sold by the different dealers

The n dealers sell a homogeneous good and post prices ordered from the lowest to the highest

$$p_1 \leq p_2 \leq \cdots \leq p_n$$

We assume that traders expect to find any of the prices with equal probabilities[1]

$$Pr(p_1) = \cdots = Pr(p_n) = 1/n$$

The price distribution is common knowledge among traders, but traders don't know whether the price charged by the next dealer will be higher or lower if they continue searching. For tractability, we also assume a trader can always go back to a previous dealer.

Traders have heterogeneous search costs, and $G(x)$ represents the share of traders with costs lower than x. We assume the following uniform distribution

$$G(x) = \begin{cases} \frac{x}{s} & \text{if } 0 \leq x \leq v - p^* \\ \frac{v-p^*}{s} & \text{if } x > v - p^* \end{cases} \tag{2}$$

where v is the value attached to the good by every trader; $p^* \equiv \sum_j p_j/n$ is the average price; and s is the density for $0 \leq x \leq v - p^*$[2].

In equilibrium each trader j stops searching and pays p_i when the expected gain from searching a price lower than p_i equals j's search costs. The equilibrium condition is therefore:

$$x_j = \sum_{k=1}^{i-1} (p_i - p_k) Pr(p_k) \tag{3}$$

where x_j is j's search costs, and $\sum_{k=1}^{i-1}(p_i - p_k)Pr(p_k)$ is the expected gain from searching a price p_k lower than p_i.

Let q_i be the quantity demanded to a dealer with price p_i. A dealer with price p_i sells to two groups of traders:

[1] This assumption leads to closed-form solutions for the demand curves. If we do not assume a uniform distribution, similar results hold - for example see Duffie, Dworczak, and Zhu (2017) - but the math would be more complex.

[2] We scale the distribution by the average (retail) price p^* to simplify the algebra. The case without scaling is in the appendix.

- Traders who randomly found p_i, despite being willing to pay a price $p_{i+1} > p_i$ (the demand of dealers with prices higher than p_i);

- Traders with search costs higher than the expected gain from searching for a price lower than p_i.

Both types of traders are represented formally in the equation below

$$q_i = q_{i+1} + \frac{1}{i}\left[G(x_{i+1}) - G(x_i)\right]$$

Using the equilibrium condition (3), the expected demand for the dealer with price p_i simplifies to (Carlson and McAfee (1983) provides the proof, but we reproduce it in the appendix)

$$q_i = \frac{v - p_i}{sn} \tag{4}$$

As expected, the demand for Dealer i depends positively on traders' valuation of the good (v), and negatively on the price Dealer i charges (p_i). The demand for a single dealer is also affected by the density of traders and the number of dealers on the market (sn).

2.4 Prices

As in Duffie, Dworczak, and Zhu (2017), traders can exit the market if the benchmark realization is below a certain threshold[3], meaning that dealers are overcharging them. If traders leave the market, demand drops and dealers need to charge a lower price. We model this behavior using the penalty parameter Δ[4]. In the next section, we will model what happens as the threshold $\bar{y}$ changes, but we focus on dealers' profits for the time being.

[3] We are assuming that traders have an incentive to penalize the dealers by exiting the market because in expectation this behavior leads to a lower price. In the appendix, we show that this is equivalent to assuming that traders particularly value the good.

[4] The penalty parameter describes, in a reduced form, the behavior of a repeated game where a trader would cease any activity with the dealer if the realization of the signal is below the threshold - for an analogous structure see Ritter and Taylor (2011). In other words, we are simplifying the behavior of the traders - who could either stop trading with a specific dealer or exit the market altogether - since it produces the same result in both cases: a reduced expected demand for the dealer.

Profits of dealer i are

$$\pi_i \equiv \left[F\left(\bar{y} - \frac{\sum_i a_i}{n}\right)(p_i - \Delta) + \left(1 - F\left(\bar{y} - \frac{\sum_i a_i}{n}\right)\right) p_i - a_i \right] q_i \quad (5)$$

where p_i is the price offered by Dealer i; $F\left(\bar{y} - \frac{\sum_i a_i}{n}\right)$ is the probability that the realization of the benchmark is below the threshold $\bar{y}$; Δ is the penalty when the signal realization is below the threshold.

Competition among dealers drives their expected profits to 0, from (5) it follows that

$$p_i = a_i + \Delta F\left(\bar{y} - \frac{\sum_i a_i}{n}\right) \quad (6)$$

and from (4), $q_i = \left[v - a_i - \Delta F\left(\bar{y} - \frac{\sum_i a_i}{n}\right)\right] / sn$, which clarifies that traders punish the dealers by exiting the market when they infer the marginal costs are below the threshold (and so dealers overcharge them).

Each dealer i sets the price p_i to minimize the penalty Δ, which yields (the proof is in the appendix):

$$\Delta = \frac{n}{f\left(\bar{y} - \frac{\sum_i a_i}{n}\right)}$$

By substituting this equation back into (6), we obtain

$$p_i = a_i + n \frac{F\left(\bar{y} - \frac{\sum_i a_i}{n}\right)}{f\left(\bar{y} - \frac{\sum_i a_i}{n}\right)} \quad (7)$$

Therefore, dealers add a mark-up on top of the cost of production to determine the price at which they are wiling to sell. In the moral hazard literature, this mark-up is known as "information rent" as it relies on dealers' information advantage.

2.5 Threshold and precision

We can now analyze traders' behavior as the threshold $\bar{y}$ changes (as in Holmström (1982)). We assume F is normally distributed. Then, from (7), as traders decrease the threshold at which they would leave the market ($\bar{y} \to -\infty$), the penalty for dealers increases ($\Delta \to +\infty$), and $F\left(\bar{y} - \frac{\sum_i a_i}{n}\right) / f\left(\bar{y} - \frac{\sum_i a_i}{n}\right) \to 0$, implying that traders achieve their first best $p_i = a_i$ in which dealers don't charge any mark-up at all.

However, traders cannot achieve $\bar{y} \to -\infty$ as production costs cannot be negative. Moreover, an external authority (e.g. the regulator) cannot achieve $\Delta \to +\infty$, as this level of regulatory fines is simply impossible.

Another way in which the outcome can be moved towards the optimal one is to reduce the noise in the benchmark. To see this, suppose that traders choose the optimal threshold level $\bar{y}$ after observing the price p_i. Traders maximize their utility

$$\max_{\bar{y}} v - p_i(a_i, \bar{y})$$

from which we obtain the following equation for the price set by each dealer i (the derivation is in the appendix):

$$p_i = a_i + n\sigma \frac{h(\zeta)}{h'(\zeta)} \tag{8}$$

where $\zeta \equiv \frac{\bar{y} - \frac{\sum_i a_i}{n}}{\sigma}$, and $h(\zeta)$ is ζ's density function.

Equation (8) is the crucial result of the model. An increase in precision reduces the noise in the benchmark fixing process (σ) and moves the outcome closer to the first best. If the noise in the benchmark fixing process is eliminated, i.e. $\sigma = 0$, then the first best can be achieved irrespectively of the level of penalties. In such a case, traders pay a price that matches the exact cost of production of each dealer.

3 Conclusions

This note provides a theoretical underpinning for many of the rules introduced in relation to benchmarks in recent years. These rules have increased precision of the benchmarks by introducing regulatory control over the benchmark fixing process and thereby reducing the possibility of benchmark manipulation.

Aquilina, Ibikunle, Mollica, and Steffen (2017) show that underlying liquidity improved in the USD interest rate swaps market following the shift to regulation of the associated benchmark (the ICE Swap Rate). Their study lends some empirical support to the results of our theoretical model.

A Appendix

A.1 Carlson and McAfee (1983) proof

First, we can write the expected gain from searching a price lower than p_i and the expected demand q_i as

$$\sum_{k=1}^{i-1}(p_i - p_k)Pr(p_k) \equiv \frac{1}{n}\left[(i-1)p_i - \sum_{k=1}^{i-1} p_k\right] \tag{9}$$

$$q_i = \sum_{k=i}^{n} \frac{1}{k}\left[G(x_{k+1}) - G(x_k)\right] \equiv \frac{1}{n}G(x_{n+1}) - \frac{1}{i}G(x_i) + \sum_{k=i+1}^{n} \frac{1}{k(k-1)}G(x_k) \tag{10}$$

Second, by induction the following equivalence holds

$$\sum_{k=i+1}^{n} \frac{1}{k(k-1)} = \frac{n-i}{ni} \tag{11}$$

Then, from (10) and the cost distribution (2)

$$q_i = \frac{v - p^*}{sn} - \frac{x_i}{si} + \sum_{k=i+1}^{n} \frac{x_k}{sk(k-1)}$$

In equilibrium, the search cost equals the expected gain from searching a lower price, then using (9), we obtain

$$
\begin{aligned}
q_i =& \frac{1}{sn}\left\{v - p^* - \frac{\left[(i-1)p_i - \sum_{j=1}^{i-1} p_j\right]}{i} + \sum_{k=i+1}^{n} \frac{\left[(k-1)p_k - \sum_{j=1}^{k-1} p_j\right]}{k(k-1)}\right\} \\
=& \frac{1}{sn}\left\{v - p^* - p_i + \frac{1}{i}p_i + \frac{\sum_{j=1}^{i-1} p_j}{i} + \sum_{k=i+1}^{n} \frac{p_k}{k} - \sum_{k=i+1}^{n}\sum_{j=1}^{k-1} \frac{p_j}{k(k-1)}\right\} \\
=& \frac{1}{sn}\left\{v - p^* - p_i + \frac{\sum_{j=1}^{i} p_j}{i} + \sum_{k=i+1}^{n} \frac{p_k}{k} - \sum_{k=i+1}^{n}\sum_{j=1}^{k-1} \frac{n-i}{ni}p_j\right\} \\
=& \frac{1}{sn}\left\{v - p^* - p_i + \frac{\sum_{j=1}^{i} p_j}{i} + \sum_{k=i+1}^{n} \frac{p_k}{k} - \sum_{k=i+1}^{n}\sum_{j=1}^{k-1} \frac{p_j}{i} + \sum_{k=i+1}^{n}\sum_{j=1}^{k-1} \frac{p_j}{n}\right\} \\
=& \frac{1}{sn}\left\{v - p^* - p_i + \sum_{j=1}^{n} \frac{p_j}{n}\right\} \\
=& \frac{1}{sn}\left\{v - p^* - p_i + p^*\right\} \\
=& \frac{v - p_i}{sn}
\end{aligned}
$$

A.2 Derivation of Δ

Dealers set the price p_i to minimize the penalty Δ. From the quantity equation (4) and the price equation (6), we obtain

$$
q_i = \frac{v - a_i - \Delta F\left(\bar{y} - \frac{\sum_i a_i}{n}\right)}{sn}
$$

Dealers know traders expect $p_i = a_i$ and, arranging the previous equation, we obtain

$$
\Delta = \frac{v - p_i - snq_i}{F\left(\bar{y} - \frac{\sum_i p_i}{n}\right)} \tag{12}
$$

So, each dealer i solves

$$
\min_{p_i} \Delta
$$

from the first order conditions it follows that

$$-1 + \frac{v - p_i - snq_i}{F\left(\bar{y} - \frac{\sum_i p_i}{n}\right)} \frac{f\left(\bar{y} - \frac{\sum_i p_i}{n}\right)}{n} = 0$$

and using (12), we finally obtain

$$\Delta = \frac{n}{f\left(\bar{y} - \frac{\sum_i p_i}{n}\right)}$$

A.3 Derivation of p_i

Traders maximize their utility

$$\max_{\bar{y}} v - p_i(a_i, \bar{y})$$

Using the price equation (6), from the first order conditions we obtain

$$n\left[1 - \frac{F\left(\bar{y} - \frac{\sum_i a_i}{n}\right) f'\left(\bar{y} - \frac{\sum_i a_i}{n}\right)}{f^2\left(\bar{y} - \frac{\sum_i a_i}{n}\right)}\right] = 0$$

from which

$$F\left(\bar{y} - \frac{\sum_i a_i}{n}\right) = \frac{f^2\left(\bar{y} - \frac{\sum_i a_i}{n}\right)}{f'\left(\bar{y} - \frac{\sum_i a_i}{n}\right)}$$

Using this result, the price equation (7) becomes

$$p_i = a_i + n\frac{f\left(\bar{y} - \frac{\sum_i a_i}{n}\right)}{f'\left(\bar{y} - \frac{\sum_i a_i}{n}\right)} \tag{13}$$

To explicit the role of precision in benchmark fixing, define $\zeta \equiv \frac{\bar{y} - \frac{\sum_i a_i}{n}}{\sigma}$, and let $h(\zeta)$ be ζ density function. Then, by changing the variable in (13), we obtain

$$p_i = a_i + n\sigma\frac{h(\zeta)}{h'(\zeta)}$$

A.4 The model without scaling

To avoid scaling the support of the distribution in (2), we need to assume that the benchmark affects each dealer with a different probability of being punished.

Without scaling the distribution, we would have

$$G(x) = \begin{cases} \frac{x}{s} & \text{if } 0 \leq x \leq v \\ \frac{v}{s} & \text{if } x > v \end{cases} \tag{14}$$

From Appendix A.1, this distribution would lead to

$$q_i = \frac{v + p^* - p_i}{sn}$$

With different probabilities for each dealer i, we would have

$$p_i = a_i + n \frac{F_i\left(\bar{y} - \frac{\sum_j a_j}{n}\right)}{f_i\left(\bar{y} - \frac{\sum_j a_j}{n}\right)}$$

Then,

$$q_i = \frac{v + \frac{\sum_{k=1}^n a_k}{n} - a_i + n\left[\sum_{k=1}^n \frac{F_k\left(\bar{y} - \frac{\sum_j a_j}{n}\right)}{n f_k\left(\bar{y} - \frac{\sum_j a_j}{n}\right)} - \frac{F_i\left(\bar{y} - \frac{\sum_j a_j}{n}\right)}{f_i\left(\bar{y} - \frac{\sum_j a_j}{n}\right)}\right]}{sn}$$

By following the steps as in A.3, we obtain

$$q_i = \frac{v + \frac{\sum_{k=1}^n a_k}{n} - a_i + n\left[\sum_{k=1}^n \frac{\sigma_k}{n} \frac{h_k(\zeta_k)}{h'_k(\zeta_k)} - \sigma_i \frac{h_i(\zeta_i)}{h'_i(\zeta_i)}\right]}{sn}$$

where $\zeta_i \equiv \frac{\bar{y} - \frac{\sum_i a_i}{n}}{\sigma_i}$.

When we do not scale the distribution, marginal costs' and noise's effects on the expected demand are with respect to the average marginal cost and noise in the market. We have inefficiencies also in this case: an efficient dealer ($\frac{\sum_{k=1}^n a_k}{n} > a_i$) may have a low demand just because noise affects him more than market average ($\sum_{k=1}^{n} \frac{\sigma_k}{n} \frac{h_k(\zeta_k)}{h'_k(\zeta_k)} < \sigma_i \frac{h_i(\zeta_i)}{h'_i(\zeta_i)}$). As in the case where we scale the distribution, an increase in precision moves the outcome closer to the first best which, in this case, is $p_i = a_i$ and $q_i = \frac{v + \frac{\sum_{k=1}^{n} a_k}{n} - a_i}{sn}$.

A.5 The model without penalty Δ

If no penalty Δ is available, then each dealer i would choose his price p_i to maximize his profits given the expected demand (while in the penalty case he chooses p_i to minimize Δ), i.e.

$$\max_{p_i} p_i q_i - a_i q_i$$

using (4)

$$\max_{p_i} (p_i - a_i) \frac{v - p_i}{sn}$$

From the first order conditions we obtain

$$p_i = \frac{v + a_i}{2}$$

Therefore, we need Δ to map the noise in benchmark fixing into the prices.

Moreover, we have seen that in case the penalty Δ is available, the price equation is

$$p_i = a_i + n\sigma \frac{h(\zeta)}{h'(\zeta)}$$

Then, traders have an incentive to punish the dealers if the price with Δ is lower than the price without it:

$$\frac{v + a_i}{2} > a_i + n\sigma \frac{h(\zeta)}{h'(\zeta)}$$

from which

$$v > a_i + 2n\sigma \frac{h(\zeta)}{h'(\zeta)}$$

Therefore, the implicit assumption is that traders highly value the good they are trading.

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