

On the representativeness of the LBMA Silver Price

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Abstract. As a reaction to the threat of market manipulation, the main benchmark for silver – the LBMA Silver Price – underwent changes in its methodology in August 2014, and became regulated in April 2015. However, unusual differences between the LBMA Silver Price and the silver spot price in the period following these changes have raised questions over a potential negative impact on the representativeness of the benchmark. In this paper, we study the benchmark from March 2017 to September 2017, analysing the effects of temporary imbalances in bid and offer volumes during the benchmark-setting auction, and associated dislocations relating to the spot market. We find no evidence that the benchmark is systematically unrepresentative in this period.

1. Introduction

The LBMA (London Bullion Market Association) Silver Price is the main benchmark for the price of silver. In July 2014, a US lawsuit alleged that the banks setting the benchmark had been manipulating the silver market since January 2007.¹ In August 2014 the LBMA Silver Benchmark moved from a phone close-auction to an electronic auction. In April 2015, the FCA began regulating the benchmark.² These process and regulatory changes aimed to improve the transparency and the robustness of the benchmark.

After the above interventions, concerns rose about differences between auction and silver spot prices, also called dislocations.³ These dislocations can be used as a measure of the representativeness of the benchmark (ideally, the difference between the LBMA Silver Price and the price of silver would be zero). Figure 1 plots over time the dislocations from March 2017 to September 2017 (left panel) and their corresponding distribution (right panel). The time series is distributed around zero and slightly skewed to the right, but also shows some extreme events, like the suspension of the auction on 10 April 2017. Because of these dislocations and suspensions, market participants started questioning the representativeness of the LBMA Silver Price, wondering whether they can still use it as a benchmark for the price of silver.⁴

Anecdotal feedback received suggests that auction participants may no longer allow clients to change orders during the auctions and that proprietary trading in the auctions has reduced. This change of behaviour may possibly be due to auction participants' concerns that revisions of bids or offers can be perceived as manipulation.⁵ We therefore want to understand whether these concerns may have led to 'stickiness' of volumes during the auction, leading to temporary imbalances (differences between bids and offers) and so to dislocations.

¹ <https://www.reuters.com/article/us-silver-fix-lawsuit/silver-bullion-banks-accused-of-manipulation-in-u-s-lawsuit-idUSKBN0FU29920140725>

² FCA PS15/06: <https://www.fca.org.uk/publications/policy-statements/ps15-6-bringing-additional-benchmarks-regulatory-and-supervisory>

³ <https://www.bulliondesk.com/silver-news/update-silver-market-disarray-after-benchmark-priced-far-below-spot-rate-108129/>

⁴ <https://www.bullionstar.com/blogs/ronan-manly/death-spiral-lbma-gold-silver-auctions/>

⁵ <https://www.reuters.com/article/us-silver-benchmark-exclusive-idUSKBN17T1XS>

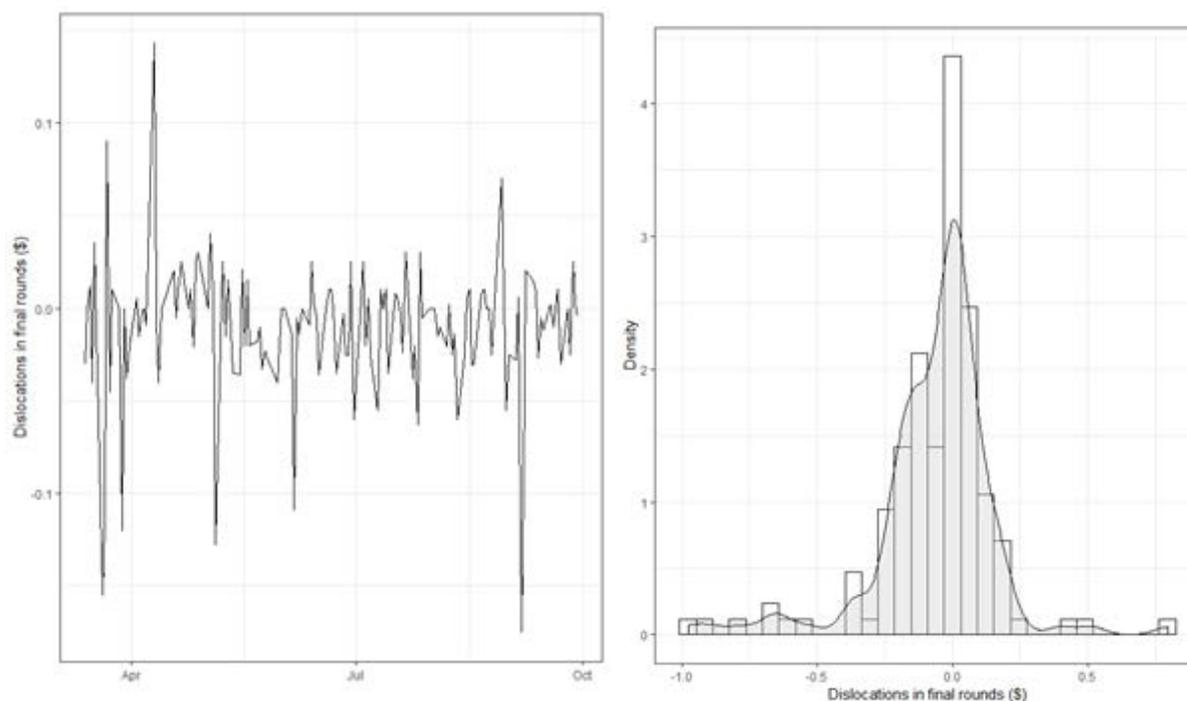


Figure 1: Differences between the LBMA Silver Price and the silver spot price over time (left panel) and their distribution (right panel).

The figure plots the dislocations in US dollars and covers the period from March 2017 to September 2017. The time series displays some extreme events, while the distribution confirms that dislocations are centred around zero.

Our data cover March to September 2017 only – ie the period after both changes had taken effect – so we cannot assess whether the imbalances are due to the decision to shift price setting to an electronic platform or the decision to bring the benchmark into the regulatory perimeter. For the same reason, we could not compare the representativeness of the benchmark pre- and post-interventions. In the paper, we therefore focus on whether temporary imbalances affect the representativeness of the benchmark.

We develop a structural vector error correction model to estimate the relationship between the variables. We then use these estimates to analyse how the price generated by the electronic auction responds to a temporary surge in imbalances. We find that the auction price incorporates the information from the spot market.

1.1 The auction

The setting of the silver price in London began in 1897 as a private auction among a few precious-metal dealers. Blagg (2014) summarises the reasons why the setting began in London:

“Branches of all the Indian and Far Eastern banks were located in London; these were the principal intermediaries for the mercantile trade of the Far East. Apart from this, geographically, London was a convenient centre for supplying the coinage requirements of European nations. Secondly, there were regular weekly shipments of silver from American and Mexican producers to London, which were dispatched to smelters and refiners before being sold to India through the London brokers [...] As

London handled the bulk of silver produced, it followed that it was London that called the tune and 'fixed' the price for the whole world."

Nowadays, the LBMA Silver Price is the equilibrium price of an electronic Walrasian auction. The auction takes place at 12pm BST each working day and opens with a price (in US dollars per troy ounce) based on live quotes from the spot market. At the opening price, participants submit their buy and sell volume orders in lakhs or quarter lakhs, ie 100,000 or 25,000 ounces (approximately 3.1 or 0.78 tonnes). Next, the auction algorithm attempts to match all orders within an acceptable difference between bid and offer volumes – the tolerance level. If the algorithm fails at matching the orders, the auction starts with a new round and a different price, based on the imbalance, at which participants can submit new bids and offers. Each round lasts 30 seconds, after which participants only observe aggregated bids and offers. The auction stops when buy and sell orders are within the tolerance level, reaching the equilibrium price.

Participation in the auction is restricted to members of LBMA only. Firms report participating in the auction for prestige, given the historical and important role of the benchmark, and because of their clients' needs. Just before the change to an electronic platform, 3 wholesale banks participated in the auction (Deutsche Bank, The Bank of Nova Scotia, HSBC). In the period under analysis, 7 banks (China Construction Bank, HSBC, JP Morgan Chase, UBS, Morgan Stanley, The Bank of Nova Scotia and The Toronto Dominion Bank) could participate but only 6 participants were active. After the period under analysis, in August 2018, 10 firms contributed to the benchmark (Coin 'N Things, Goldman Sachs, HSBC, INTL FC Stone, Jane Street Global Trading, JP Morgan Chase, Koch Supply and Trading, Morgan Stanley, The Bank of Nova Scotia and The Toronto Dominion Bank). Firms excluded from the auction can still trade silver in a spot market, which is also active during the auction.

The first co-administrators of the electronic auction were Thomson Reuters and the CME Group. They announced their decision to step down on March 2017, despite being under contract until 2019. They cited changes to the European Benchmark Regulation as a justification for stepping down.⁶ Intercontinental Exchange (ICE) became the new benchmark administrator on October 2017.

2. Data and summary statistics

We observe spot prices (at the start of each round), auction prices, aggregated bid and offer volumes, and the number of participants at each round in each auction between March 2017 and September 2017. During this period, Thomson Reuters and the CME Group increased the tolerance level from 3 to 5 units on 23 March 2017.⁷ In total, we observe 321 rounds in 139 auctions.

Table 1 shows that the median of the dislocations in all the 321 observations equals zero, as for the dislocations in the final round plotted in Figure 1. The table also suggests that offer volumes tend to be higher than bid volumes, explaining the skewness of the distribution plotted in Figure 1. Most auctions had at least 5 participants and lasted for 3 rounds.

Table 2 shows that, on 10 April, when the auction was suspended, the offer volumes were extremely low while bid volumes were very high, with none of the 2 sides of the market adjusting their volumes during the auction. The imbalances dropped from -15.12, when the auction got suspended, to -5.62 when it re-opened, eventually leading to the successful end of the auction.⁸

⁶ <https://uk.reuters.com/article/uk-silver-benchmark-lbma-idUKKBN18Z1DZ>

⁷If we restrict the analysis to the period when the tolerance level is 5 units, avoiding possible structural breaks, our conclusions do not change.

⁸ We are not aware of any events that occurred during the suspension.

Table 1: Summary statistics of the LBMA Silver Price auctions

Descriptive statistics calculated over 321 rounds in all auctions between March 2017 and September 2017. Dislocation is the difference between the auction and spot price; imbalances is the difference between offer and bid volumes.

	Min.	1 st Q	Median	Mean	3 rd Q	Max
Dislocations	-0.175	-0.01	0	-0.008	0.004	0.143
Imbalances	-29.370	-0.982	3.48	2.845	6.282	28.1
Bid volumes	0	3.895	7.270	8.938	11.728	56.83
Offer volumes	0.53	7.19	10.65	11.78	14.42	57.55
#Participants	3	5	6	5.355	6	6
Rounds	1	1	2	2.329	3	16

Table 2: The LBMA Silver Price Auction on 10 April 2017

Detailed data on the Silver Price Auction on 10 April 2017. The auction got suspended after 6 rounds and, after 17 minutes, the auction restarted. Participants adjusted their bids and offers, and the auction concluded after 2 additional rounds. Dislocation is the difference between the auction and spot price, Imbalances is the difference between offer and bid volumes.

Round Number	Participants	Spot Price	Auction Price	Offer Volumes	Bid Volumes	Imbalances	Dislocation
1	6	17.9015	17.90	1.25	30.62	-29.37	-0.0015
2	6	17.8850	17.95	3.45	28.82	-25.37	0.0650
3	6	17.9100	18	6.45	26.07	-19.62	0.0900
4	6	17.9200	18.03	6.45	24.57	-18.12	0.1100
5	6	17.9250	18.06	6.45	22.57	-16.12	0.1350
6	6	17.9470	18.09	6.45	21.57	-15.12	0.1430
1	6	17.9335	17.92	9.75	15.37	-5.62	-0.0035
2	6	17.9300	17.94	10.45	14.07	-3.62	0.0100

Table 2 also shows that 6 firms participated in each round, while the spot price changed by about \$0.03 (0.16%) between the start and the end of the auction. So, neither the number of auction participants nor the spot price are likely to have triggered the suspension.

When looking at other major dislocation events (those with a difference between auction and spot price higher than \$0.10), it seems that either bids or offers were particularly 'sticky' and that the successful end of an auction relied on adjustments made by only one side of the market (Table 3).

Tables 2 and 3 suggest that, in few occasions, a certain stickiness of one side of the market may have led to temporary imbalances. In the next section, we study the effects of these temporary imbalances, including any implications for the representativeness of the benchmark.

Table 3: Initial and final volumes when main price dislocations happened

Summary statistics of auctions with major price dislocations, ie when the differences between auction and spot price was larger than \$0.10. The auction on 10 April 2017 was suspended after 6 rounds and concluded after 2 additional rounds.

Date	Total Number of Rounds	Initial Bid Volumes	Final Bid Volumes	Initial Offer Volumes	Final Offer Volumes
2017-03-20	16	2.41	2.41	8.01	5.66
2017-03-21	9	5.72	7.47	12.38	9.43
2017-03-28	6	2.99	2.74	16.09	7.19
2017-04-10	6 (+2)	30.62	21.57	1.25	6.45
2017-05-05	8	8.35	20.80	21.63	24.93
2017-06-06	11	3.62	7.62	16.00	12.00
2017-09-06	6	2.82	17.82	27.44	21.44

3. A SVEC representation of the LBMA Silver Price Auction

a. The methodology

We develop a structural vector error correction (SVEC) model with 4 variables: auction price, spot price, aggregated bid volumes and aggregated offer volumes. We use the model to understand the effects of a temporary surge in imbalances within the auction.

A spot market is open while the auction takes place. And the silver in the auction is like the silver in the spot market. So, when modelling the auction, we need to consider that (because of arbitrage opportunities) spot market prices may influence bids and offers; bids and offers may influence auction prices; and auction prices may influence bids and offers. A SVEC model allows us to study these interdependencies: we can model the relation between volumes and prices as in structural vector autoregressive (SVAR) models, while analysing price discovery in multiple markets as in vector error correction (VEC) models.

Both SVAR and VEC models have already been applied in market microstructure separately. For example, Hasbrouck (1991) was the first to suggest applying SVAR to analyse relationships between volume and price, and Eaves and Williams (2007) apply a SVAR model to the Walrasian auction on the Tokyo Grain Exchange, which is analogous to the LBMA Silver Price auction. Hasbrouck (1995) develops a VEC model to identify the incorporation of new information into prices for homogenous securities that are traded in multiple markets, which is analogous to our framework where silver is traded in the auction and in the spot market simultaneously.

Our approach combines the SVAR and VEC models used in the literature in a single model. For robustness, we also estimate a SVAR and a VEC model analogous to the ones in the literature (see Appendix).

We use the SVEC model to estimate the short- and long-run relationship between the variables. We then use these estimates to analyse how the auction responds to a temporary surge in imbalances (by plotting the impulse response functions).

b. The model

A standard SVEC model has the following form

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + B u_t \quad (1)$$

Where y_t is a K -dimensional vector and Δy_t is its first difference; Γ_i is an $(K \times K)$ matrix of coefficients with $i \in \{1, 2, \dots\}$; and $\Pi = ZA'$ is a matrix of coefficients with Z as $(K \times r)$ matrix of real numbers and A' an $(r \times K)$ matrix of real numbers whose rows are linearly independent cointegrating vectors; B is an $(K \times K)$ matrix which represents the structural innovations; and u_t is a stationary K -dimensional vector of error terms.

In our model, the vector y_t has dimension $K = 4$ and includes (in this order): spot price, auction price, bid volumes and offer volumes. The Dickey and Fuller test confirms the existence of a unit root in the auction and spot price series.

The Beveridge-Nelson representation of the model tells us that the number of stationary cointegration relationships, r , equals the number of shocks with temporary effects; while $K - r$ shocks have permanent effects, see Beveridge and Nelson (1981). The Johansen procedure (trace test) permits to test more than 1 cointegration relationship among time series. The test suggests that 3 variables are cointegrated, ie $r = 3$. Since $K = 4$, our model contains 3 temporary shocks and 1 permanent shock.

We used the entire series of 321 observations as a single timeseries, since estimating a model within a single auction is unfeasible because most of the auctions had 3 rounds, as seen in Table 1. We introduce 2 dummy variables to control for differences between within-auction and between-auction dynamics: one dummy indicates the initial rounds, while the other dummy indicates the final rounds. We also control for the number of participants, but this variable does not affect the estimation. Additionally, we treat the observations before and after the suspension on 10 April as if they were 2 different auctions as, during the suspension, participants may have received new information.

The Akaike information criterion and the Schwarz Criterion suggest the introduction of only 1 lag. When we estimate the model with only 1 lag, the ARCH-LM test and the Portmanteau Test suggest the residuals are homoscedastic and not autocorrelated. Therefore, the SVEC model we estimate is

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \beta_1 \delta_{\text{first rounds}} + \beta_2 \delta_{\text{final rounds}} + B u_t$$

where $\delta_{\text{first rounds}}$ and $\delta_{\text{final rounds}}$ are the dummies and β_i are their coefficients.

c. Restrictions and estimations

We use the SVEC model to estimate the short- and long-term relation among the 4 variables in our model. See Engle and Granger (1987) for how to derive the long-term relations. To identify our SVEC model, we need to impose some restrictions on these relations.

Table 4 reports the estimation of the short-term relations among the variables, from which we can also observe the restrictions imposed. The first 2 rows of Table 4 imply that a shock to bid and offer volumes does not have an immediate impact on the spot and auction price. This assumption follows from the structure of the auction, as volumes are submitted after prices are revealed. In the second row, we impose that a shock on the spot price does not immediately influence the auction price. This

Table 4: Estimated coefficients of the contemporaneous impact matrix

Coefficients of the contemporaneous impact matrix which represents matrix B in the SVEC model $\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \beta_1 \delta_{first\ rounds} + \beta_2 \delta_{final\ rounds} + B u_t$. The structure of the matrix shows the restrictions imposed: a shock to the spot price, $\varepsilon_t^{spot\ price}$, cannot immediately influence the auction price; a shock to the volumes cannot immediately influence the prices; offer volumes cannot immediately influence bids, and viceversa. The assumptions follow from the structure of the auction.

Contemporaneous impact matrix	$\varepsilon_t^{spot\ price}$	$\varepsilon_t^{auction\ price}$	$\varepsilon_t^{bid\ volumes}$	$\varepsilon_t^{offer\ volumes}$
Spot price	0.023	0.131	0	0
Auction price	0	0.137	0	0
Bid volumes	-0.498	-0.47	4.533	0
Offer volumes	0.546	0.263	0	4.143

Table 5: Estimated coefficients of the long-run impact matrix

Coefficients of the long-run impact matrix of the SVEC model $\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \beta_1 \delta_{first\ rounds} + \beta_2 \delta_{final\ rounds} + B u_t$.

Long-run impact matrix	$\varepsilon_t^{spot\ price}$	$\varepsilon_t^{auction\ price}$	$\varepsilon_t^{bid\ volumes}$	$\varepsilon_t^{offer\ volumes}$
Spot price	0.032	0.124	-0.002	-0.011
Auction price	0.032	0.124	-0.002	-0.011
Bid volumes	-0.008	-0.032	0.0005	0.003
Offer volumes	-0.025	-0.097	0.001	0.008

assumption also follows from the structure of the auction, as once the auction price is fixed, it relies on bids and offers only.

The third and fourth row of Table 4 imply that offers do not simultaneously influence bids, and vice versa. Again, this assumption is due to the structure of the auction, as bids and offers are submitted at the same time. Table 5 reports the estimation of the long-term relationship among the variables and we did not impose any restrictions, since we want to understand how a temporary shock affects this relationship.

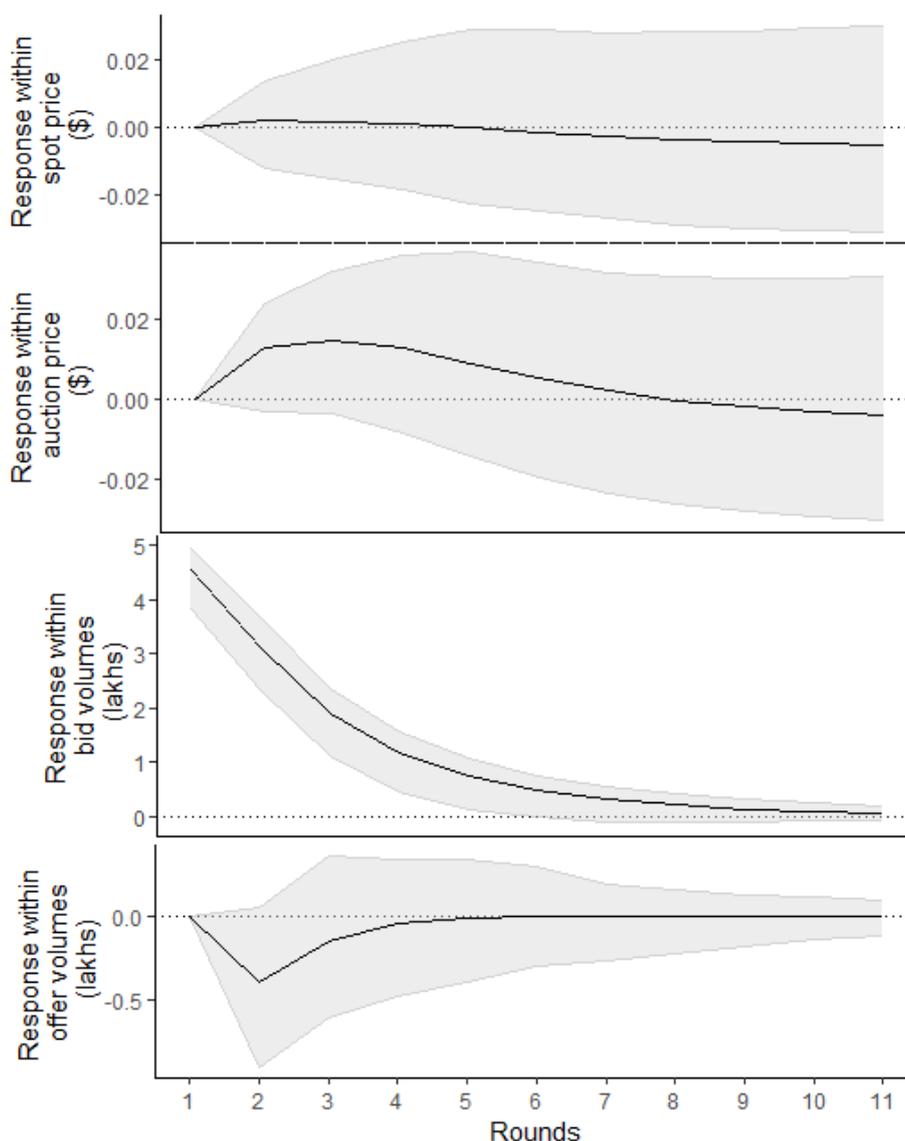


Figure 2: Dynamics of the auction after a shock to bid volumes

Impulse responses within spot price, auction price, bid volumes and offer volumes from a shock to bid volumes in round 1. The dynamics are as implied by the estimated coefficients in Tables 5 and 6. The grey area represents the 95% confidence interval.

The restrictions imposed on Table 4 may seem too restrictive despite being derived from the structure of the auctions. We provide robustness checks in the appendix, confirming our results. We now use the estimates in Table 4 and Table 5 to plot the dynamics of the LBMA Silver auction.

d. Dynamics

In the previous sections, we have shown that price dislocations may be due to temporarily high imbalances between bid and offer volumes. From the estimates of our model, we can study the effect of a temporary shock in volumes and see how participants behave and prices evolve. Figure 2

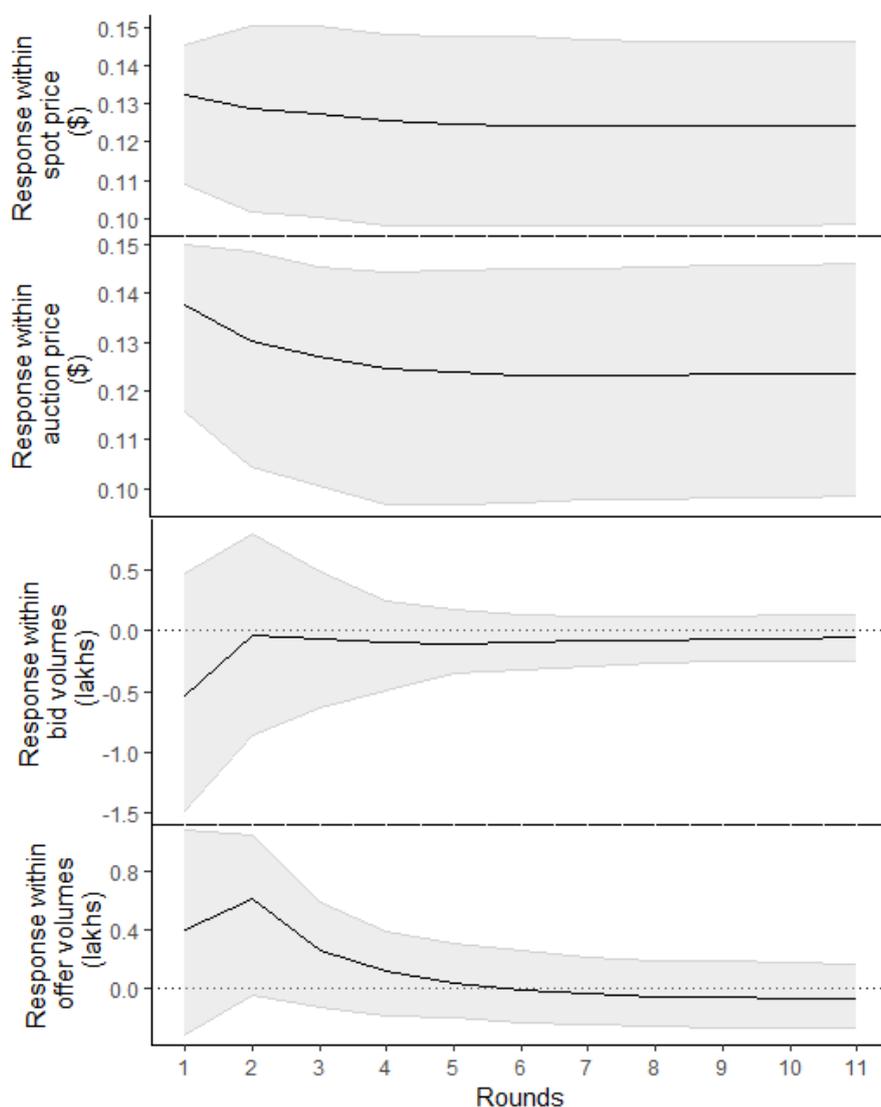


Figure 3: Dynamics of the auction after a shock to auction price

Impulse responses within spot price, auction price, bid volumes and offer volumes from a shock to auction price in round 1. The dynamics are as implied by the estimated coefficients in Tables 5 and 6. The grey area represents the 95% confidence interval.

plots the effects of a shock in bid volumes on the auction dynamic (a shock to offers shows similar results).

The dynamics in Figure 2 display stickiness of offer volumes, ie offer volumes do not adjust to a surge in bid volumes. However, the figure also shows that bids do adjust and progressively reduce. In the long run, the auction price converges towards the spot one as bids decrease.

From our data, we cannot infer whether the imbalances led to dislocations, or the algorithm adjusting the auction price led to the imbalances. Figure 2 only represents the effects of a shock to

volumes. So, in Figure 3, we plot the shock due to the setting algorithm, which we model as a shock to the auction price.

The dynamics in Figure 3 suggest that the auction price still converges towards the spot price. In the first round the spot price integrates only part of the shock, suggesting that revisions are more frequent in the auction than in the spot market. Volumes react to the difference between auction and spot price, and revert to their initial levels as the price dislocation reduces.

Overall, both dynamics suggest that auction participants react to differences in auction and spot market prices. Even if only one side of the market may revise its quotes, the revision permits the auction price to integrate the information from the spot market.

4. Conclusions

Our analysis suggests that price dislocations, regardless whether they are due to temporary imbalances or to the setting algorithm, did not systematically compromise the representativeness of the LBMA Silver Price benchmark. However, because we only analysed data after the introduction of the electronic platform and the regulation of the benchmark, we cannot say whether representativeness deteriorated.

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Appendix – Robustness

In this appendix, we assess the robustness of our assumptions about the structure of matrix B in Equation 1. First, we estimate a simple SVAR model showing that imbalances react to price dislocations. Then, we estimate a VEC model and show that the auction price incorporates the information from the spot market.

a. Verifying auction participants' reaction

As in Eaves and Williams (2007), we now estimate a simpler SVAR model with price dislocations and imbalances. First, we confirm the absence of unit roots, and choose the lag length using the Akaike information criterion and the Schwarz Criterion, which suggest that the optimal lag is 1.

Then, we model the auction as follows

$$\begin{pmatrix} \gamma_{11} & 0 \\ \gamma_{21} & \gamma_{22} \end{pmatrix} \begin{pmatrix} PD_t \\ I_t \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} PD_{t-1} \\ I_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \quad (2)$$

Where PD_t is the price dislocation at time t ; I_t is the imbalance at time t ; and the right-hand side is the canonical representation of an OLS regression with a_i being the fixed effects, β_i the coefficients and ϵ_i the error terms. The γ -matrix on the left-hand side represents the contemporaneous relation between the 2 variables. The effect of Imbalances on current price dislocations is set to 0 as the auction price is revealed before the submission of bids and offers.

Figure 4 plots the effects of a price dislocation shock on dislocations and imbalances. The shock, which increases the dislocations, initially decreases the imbalances. Then the situation reverts and in the long run the shock is absorbed. Also, in our previous model, we have observed that bid and offer volumes may react differently, leading to changes in the imbalances before the shock is absorbed. These results confirm our initial finding that auction participants react to changes in auction and spot prices.

b. Verifying price discovery

In the previous section, we have also seen that the auction price incorporates the information from the spot market. To verify this result, we follow Hasbrouck (1995) and model the 2 (cointegrated) price series as the following system of equations (a VEC model)

$$\Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \dots + \Gamma_{p-1} \Delta x_{t-p+1} + u_t \quad (3)$$

Where $x_t = \begin{pmatrix} P_{\text{auction},t} \\ P_{\text{spot},t} \end{pmatrix}$ is the vector of prices and Δx_t is its first difference; Π is the error correction term; Γ_i is the coefficient, and u_i the error term. Also, in this case, the Akaike information criterion and the Schwarz Criterion suggest the introduction of only one lag.

Figure 5 plots how a shock to the auction price is immediately reflected into a change of the spot price. Despite prices converge in the long run, less of the shock is incorporated into the spot market as the initially curve is less steep in the spot price than in the auction price. The situation is different when the spot price faces a shock, as plotted in Figure 6. The auction price does not immediately react to the change, but it converges to the spot price fully incorporating the shock.

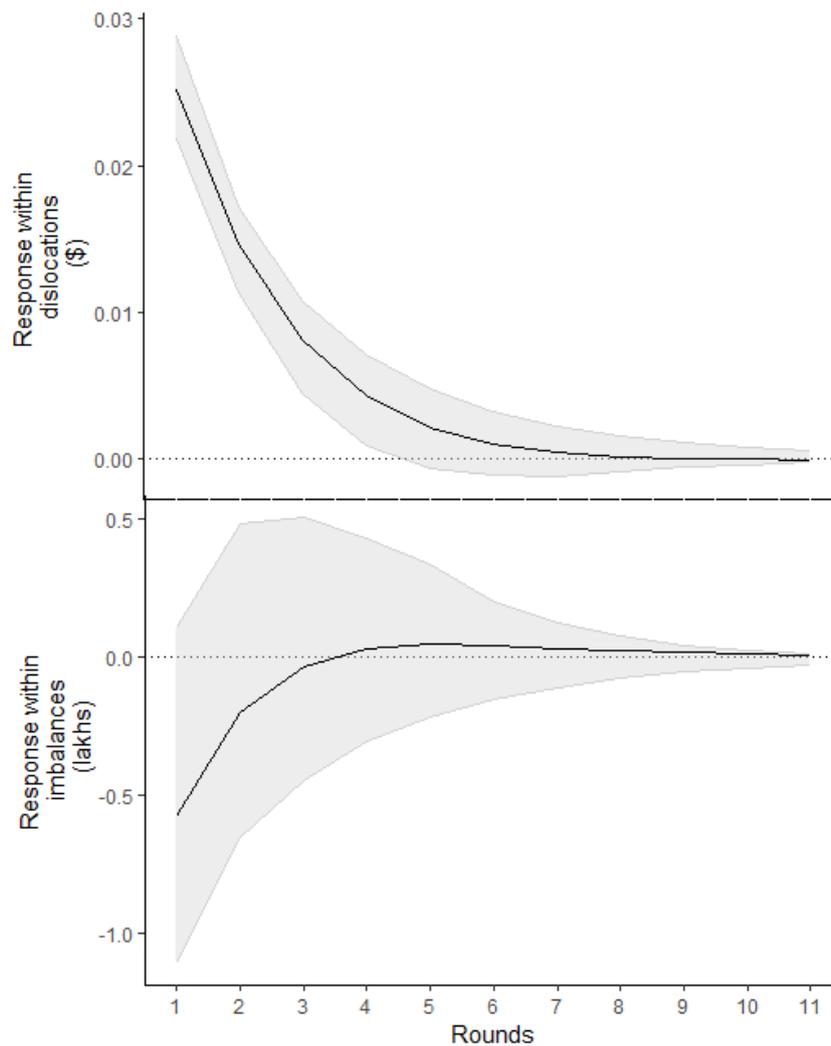


Figure 4: Dynamics of the auction after a shock to dislocations.

Impulse responses within dislocations and imbalances from a shock to dislocations in round 1. The dynamics are as implied by the estimated coefficients from the SVAR model in Equation 2. The grey area represents the 95% confidence interval.

These results may suggest a greater frequency of errors in the auction. Overall, the information in the spot market is well incorporated into the price of the auctions, confirming the results of the SVEC analysis.

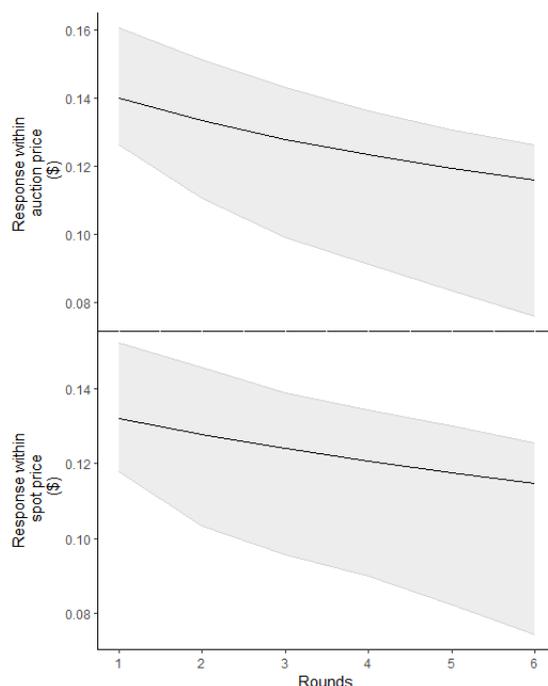


Figure 5: Dynamics of auction and spot prices after a shock to auction price.

Impulse responses within auction and spot price from a shock to auction price in round 1. The dynamics are as implied by the estimated coefficients from the VEC model in Equation 3. The grey area represents the 95% confidence interval.

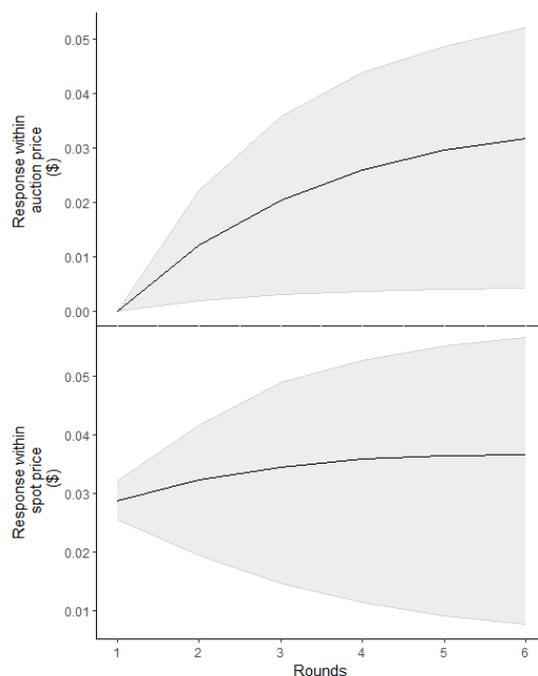


Figure 6: Dynamics of auction and spot prices after a shock to spot price.

Impulse responses within auction and spot price from a shock to spot price in round 1. The dynamics are as implied by the estimated coefficients from the VEC model in Equation 3. The grey area represents the 95% confidence interval.